THE SEPARATION EFFECT AT THE OUTLET OF A VESSEL WITH A MECHANICALLY AGITATED SUSPENSION

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Reactions in solid–liquid systems are often carried out in agitated vessels. For continuous flow operations the suspension is usually withdrawn by an overflow from the liquid level or in multistage column reactors from the bottom of the column. The movement of the liquid inside the vessel is determined by the stirrer. At the point of outlet the liquid has to change its direction and velocity. The solid particles follow the path of the liquid with a certain amount of inertia and the concentration of solid particles at the outlet of the reactor in general differs from the average concentration of solid particles in the vessel. The ratio of these two concentrations is called the separation coefficient. In order to calculate the performance of such a reactor the separation coefficient has to be known.

Mattern, Bilous and Piret[1] measured the separation coefficient in a stirred vessel with an overflow from the liquid level. Rushton[2] tried to describe this effect quantitatively. For measuring the separation coefficient he used a simplified geometry of the outlet, the so called "isokinetic arrangement" where the outlet stream from the vessel does not change its direction but only its velocity. The arrangement according to Rushton is shown on Fig. 1. For calculating the separation coefficient Rushton used empirical equations which are valid only for the system water–glass and water–sand.

This contribution is an attempt to theoretically describe the separation effect for the isokinetic withdrawal from a stirred vessel. A theoretical treatment of the simple isokinetic withdrawal of a suspension could in the future be used as a guide for determining semi-empirical correlations for calculating the separation coefficients for suspensions withdrawn from the more geometrically complicated outlets from an overflow or from the bottom of vessels as employed in the industry.

THEORETICAL

Let us first of all consider the case where the linear velocity of the suspension in the outlet is higher than the linear velocity inside the vessel.

Similarly to Rushton we assume an isokinetic arrangement and a homogenous suspension inside the vessel. With these assumptions we can describe the outlet of the suspension simply as flow from a pipe of larger diameter D_i to an outlet opening of diameter D_e. D_i is a hypothetical diameter of the suspension stream inside the vessel as it approaches the outlet opening. It can be calculated from the equation of continuity as

$$D_i = D_e (u_e/u_i)^{1/2} . \qquad (1)$$

If we further assume that the paths of the liquid and solid particles are straight lines, we can draw a simplified diagram of the flow paths as shown in Fig. 2. The full lines are the flow paths of the liquid and the dotted lines are flow paths of the solid particles. The solid particles follow the liquid paths with inertia. A certain part of the suspension stream does not enter the outlet opening. If we denote by x the radial distance of a solid particle from the wall of the hypothetical pipe, then according to Fig. 2 the condition for the solid particle to reach the outlet opening can be written as

$$x/\alpha \geq \Delta s\,, \tag{2}$$

where

$$\alpha = D_i/D_e = (u_e/u_i)^{1/2}\,. \tag{3}$$

For the width of the annulus X which contains particles that do not enter the outlet

$$X/\alpha = \Delta s\,, \tag{4}$$

so that for calculating the separation coefficient we can use the equation

$$\varphi = c_e/c_i = [(D_i - 2X)/D_i]^2 = [1 - 2X/D_e(u_e/u_i)^{1/2}]^2\,. \tag{5}$$

In calculating Δs we can use the equation for the movement of a solid spherical particle as employed by Badzioch[3]. If we furthermore consider that the resistance to particle movement obeys Newton's law and that the effect of gravity can be neglected we obtain the expression

$$\mathrm{d}v_p/\mathrm{d}t = \psi(3/4)\,[\varrho_k/(\varrho_p - \varrho_k)\,d_p]\,(v_p - v_k)^2\,. \tag{6}$$

According to Hinze[4] the friction factor ψ in a turbulent medium and suspensions of small concentration depends on the ratio of the particle diameter to the characteristic scale of turbulence. According to this theory we introduce the simplifying assumption that the friction factor can be. expressed as

$$\psi = k_1(d_p/D_e)\,, \tag{7}$$

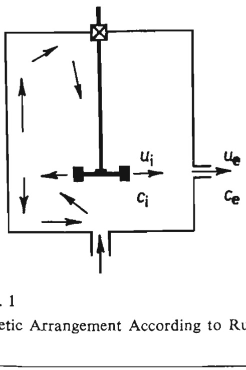

FIG. 1

Isokinetic Arrangement According to Rushton

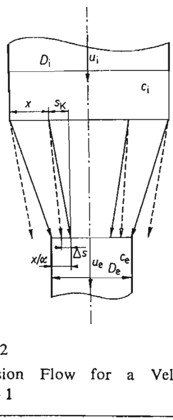

FIG. 2

Suspension Flow for a Velocity Ratio $u_e/u_i > 1$

where the constant k_1 is a function only of the geometry of the system and the outlet diameter has been chosen as the characteristic scale of turbulence. By using Eq. (7) we can rewrite Eq. (6) as

$$\mathrm{d}v_p/\mathrm{d}t = k(v_p - v_k)^2\,, \tag{8}$$

where

$$k = k_1\, 3\varrho_k/4D_e(\varrho_p - \varrho_k)\,. \tag{9}$$

Assuming that the radial liquid velocity is constant and that the radial solid particle velocity changes from 0 to v_p we can integrate Eq. (8) to give

$$v_p = v_k[1 - 1/(kv_kt + 1)]\,. \tag{10}$$

The solid particle path can then be calculated by integrating Eq. (10)

$$s_p = v_kt - [(1/k)\ln(kv_kt + 1)]\,. \tag{11}$$

Eq. (11) can further be rewritten as

$$s_k - s_p = (1/k)\ln(ks_k + 1)\,. \tag{12}$$

The liquid path s_k is a function of the annulus width X. From Fig. 2 it can be seen that this function is

$$s_k = (D_i/2) - X - (D_e/2) - (X/\alpha)\,. \tag{13}$$

By a combination of Eq. (3), (12) and (13) and substitution into Eq. (4) we obtain after rearrangement and expression for the annulus width

$$X/(u_e/u_i)^{1/2} = (1/k)\ln(k\{(D_e/2)\,[(u_e/u_i)^{1/2} - 1] + X[(u_i/u_e)^{1/2} - 1]\} + 1)\,. \tag{14}$$

If we know the value of the constant k, we can solve Eq. (14) by trial and error. The value of X thus calculated is substituted into Eq. (5) and the separation coefficient is determined.

In case the linear velocity of the suspension in the outlet is lower than the linear velocity inside the vessel, the flow between the two pipes is more complicated. A part of the liquid stream flows past the wider pipe and we must make assumptions to what extent are the liquid flow paths influenced by the exit.

The outlet of diameter D_e will contain all particles originally present outside the diameter D_i. If we denote the width of the annulus Y as that part from which the particles enter the outlet tube, we can write for the separation coefficient

$$\varphi = c_e/c_i = [(D_i + 2Y)/D_i]^2 = [1 + 2Y/D_e(u_e/u_i)^{1/2}]^2\,. \tag{15}$$

By similar arguments as in the previous case we can derive from the geometry of the flow paths that

$$s_k = D_e/2 + Y/\alpha - D_i/2 + Y\,, \tag{16}$$

which applies for the case where the liquid in the annulus of original width Y is retarded at the same rate as the liquid leaving the vessel. Then

$$Y/(u_e/u_i)^{1/2} = (1/k)\ln(k\{(D_e/2)\,[1 - (u_e/u_i)^{1/2}] + Y[(u_i/u_e)^{1/2} - 1]\} + 1)\,. \tag{17}$$

DISCUSSION

From the previous theoretical considerations it follows that the separation coefficient depends on the geometry of the system, density of liquid and solid particles, diameter of the outlet and the ratio of velocities u_e/u_i. In the limiting case when both velocities are equal the value of the separation coefficient is equal to one.

If the assumption regarding the value of the friction factor as given in Eq. *(7)* is correct, the separation coefficient should not depend on the diameter of the solid particles. All the previous considerations can be valid only for systems where the diameter of the solid particles in comparison to the outlet diameter is sufficiently small so that no separating effects due to the impact of the particles on the outlet tube wall need be considered. In deriving the theoretical equations we introduced several simplifying assumptions. The assumption about the straight line path of the solid particles and the value of the friction factor are questionable. Eq. *(6)* could in theory also be solved for a curved path of the particles. The dependency of the friction factor on the relative velocities of the liquid and solid particles could also be taken into consideration. The solution would then have to be carried out numerically in a similar manner as that used by Vitols[5] in calculating the separation coefficient for the system gas–liquid. The solution is, however, very time consuming and all the assumptions cannot be eliminated anyhow. In the experimental part of this work it will be shown that the simplified model is adequate for calculating values of the separation coefficient within reasonable accuracy.

LIST OF SYMBOLS

c_e solid particle concentration at the vessel outlet (ML^{-3})
c_i solid particle concentration inside the vessel (ML^{-3})
D vessel diameter (L)
D_i hypothetical diameter of the entering stream of suspension defined by Eq. *(1)* (L)
D_e diameter of outlet (L)
d_p diameter of solid particles (L)
k constant in Eq. *(8)* (L^{-1})
k_1 constant in Eq. *(7)*
s_k radial component of the liquid path (L)
s_p radial component of the solid particles path (L)
Δs difference of radial components of the liquid and solid particles path (L)
t time (t)
u_e linear velocity of the suspension in the outlet (Lt^{-1})
u_i linear velocity of the suspension inside the vessel in the vicinity of the outlet (Lt^{-1})
v_k radial component of the liquid velocity (Lt^{-1})
v_p radial component of the solid particles velocity (Lt^{-1})
X width of annulus containing particles which do not enter into the outlet (L)
x radial distance of the solid particles according to Fig. 2 (L)
Y width of annulus containing particles which enter the outlet (L)
y radial distance of the solid particle according to Fig. 3 (L)
α coefficient defined by Eq. (3)
ϱ_k liquid density (ML^{-3})
ϱ_p solid particle density (ML^{-3})
φ separation coefficient
ψ friction factor

REFERENCES

1. Mattern R. V., Bilous O., Piret E. L.: A.I.CH.E.J. *3*, 437 (1957).
2. Rushton J. H.: A.I.CH.E. — I. Chem. Eng. Joint Meeting, London, June 1965.
3. Badzioch S.: Brit. J. Appl. Phys. *10*, 26 (1959).
4. Hinze J. O.: *Turbulence*. McGraw-Hill, New York 1959.
5. Vitols V.: *Thesis*. University of Michigan 1964.

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DETERMINATION OF THE SEPARATION EFFECT IN A STIRRED VESSEL WITH A SUSPENSION BY A DYNAMIC METHOD

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In a previous paper[1] equations were derived for the value of the separation coefficient (ratio of the solid concentration in the outlet to the average solids concentration in the vessel) for the isokinetic withdrawal of a suspension from a stirred vessel. These are

$$\varphi = [1 - 2X/D_e(u_e/u_i)^{1/2}]^2 , \quad u_e/u_i > 1 \tag{1}$$

where X is determined from

$$X/(u_e/u_i)^{1/2} = (1/k)\,[(\varrho_p - \varrho_k)\,D_e/\varrho_k]\ln\,([k\varrho_k/D_e(\varrho_p - \varrho_k)]\,.\\ .\,\{(D_e/2)\,[(u_e/u_i)^{1/2} - 1] + X[(u_i/u_e)^{1/2} - 1]\} + 1) \tag{2}$$

and

$$\varphi = [1 + 2Y/D_e(u_e/u_i)^{1/2}]^2 , \tag{3}$$

where Y is determined from

$$Y/(u_e/u_i)^{1/2} = (1/k)\,[(\varrho_p - \varrho_k)\,D_e/\varrho_k]\ln\,([k\varrho_k/D_e(\varrho_p - \varrho_k)]\,.\\ .\,\{(D_e/2)\,[1 - u_e/u_i)^{1/2}] + Y[(u_i/u_e)^{1/2} - 1]\} + 1)\,. \tag{4}$$

The constant k in Eq. (*2*) and (*4*) depends only on the geometry of the system. The aim of this work was to verify the validity of relations (*1*)–(*4*) and to determine the value of the constant for various physical systems and a given geometry. The experiments were performed with only one geometry of the system. The independent variables were: stirrer speed, volumetric flow rate